NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2013

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1) $(x+2)^2 = 3x(x-2)$

$x^2 + 4x + 4 = 3x^2 - 6x$ M (Expanding) A

$2x^2 - 10x - 4 = 0$ A

$x^2 - 5x - 2 = 0$

$x = \frac{5 \pm \sqrt{25+8}}{2}$ M (Sub into Quadratic formula)

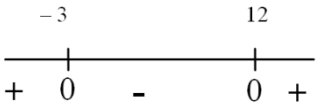
$= 5,4$ or $-0,4$ A

Answer only : Full marks
P1D (penalty for decimals) (5)

(2) $x^2 - 9x \geq 36$

$x^2 - 9x - 36 \geq 0$

$(x-12)(x+3) \geq 0$ A



M (Critical values) A

$x \leq -3$ or $x \geq 12$ CA

Must include =
$-3 \geq x \geq 12$ invalid (4)

(3) $3^x - 3^{x-2} = 72$

$3^x\left(1 - 3^{-2}\right) = 72$ M (Factorising)

$3^x\left(\frac{8}{9}\right) = 72$ A

$3^x = 72 \times \frac{9}{8}$

$= 3^4$ A

$x = 4$ CA (4)

(b) $(2m-3)(n+5) = 0$

(1) $(2 \times 1 - 3)(n+5) = 0$

$n = -5$ A (1)

(2) $m = \frac{3}{2}$ A (1)

(3) $m \in R$ AA Every number m is a solution. (2)

[17]

QUESTION 2

(a) $\sum_{k=2}^{6} \frac{2^{k-1}}{k}$

$= \frac{2}{2} + \frac{4}{3} + \frac{8}{4} + \frac{16}{5} + \frac{32}{6}$ M (Expansion)A (5 terms)

$= \frac{193}{15}$ (12,9) A

Answer only : Full marks
P1D (3)

(b) (1) $T_{12} = ar^{11}$
$= 27 \times 2^{11}$ M (Sub in Tn of G.P.)
$= 55\ 296$ A

(2)

(2) $S_{12} = \dfrac{a\left(r^{12}-1\right)}{r-1}$
$= \dfrac{27\left(2^{12}-1\right)}{2-1}$ M (Sub in Sn of G.S.)
$= 110\ 565$ A
Earnings $= \text{R}110\ 565 \times 0,5$ M
$\approx \text{R}553$ CA

ALTERNATIVELY
Earnings from G.S. with $a = \dfrac{27}{2}$ M
$S_{12} = \dfrac{\frac{27}{2}\left(2^{12}-1\right)}{2-1}$ M
$= 55282,5$ cents A
$\approx \text{R}553$ CA

(4)

(c) (1) A.P. $a = 1 \quad d = 0,75$ A
$T_9 = a + 8d$
$= 1 + 8(0,75)$ M (Sub in Tn of A.P.)
$= 7$ km CA

d = 750m & T_9 = 7000m fine

(3)

(2) $1 + (n-1)(0,75) = 10$ M (Setting T_n = 10)
$(n-1)(0,75) = 9$
$n - 1 = 12$
$n = 13$
i.e. 13th Sunday A

(2)

(3) $S_{13} = \dfrac{13}{2}\left[2 \times 1 + 12 \times 0,75\right]$ M (Sub in S_n of A.S.)
$= 71,5$ A
Total for 24 weeks $= 71,5 + 10 \times 11$
M (Ans from S13 + 10 ×(24-Ans from (2)) CA
$= 181,5$ km

(4)

ALTERNATIVELY
$S_{12} = \dfrac{12}{2}\left[2 \times 1 + 11 \times 0,75\right]$ M (Sub in S_n of A.S.)
$= 61,5$ A
Total $= 61,5 + 10 \times 12$ M CA
$= 181,5$

[18]

QUESTION 3

(a) $f(x) = 6x^2$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{6(x+h)^2-6x^2}{h}$$ A

$$= \lim_{h\to 0}\frac{6}{h}\left[x^2+2hx+h^2-x^2\right]$$ M (expanding)

$$= \lim_{h\to 0}\frac{6h}{h}\left[2x+h\right]$$

$$= \lim_{h\to 0}\left(12x+6h\right)$$ A

$$= 12x$$ CA

P1N (penalty for notation)
Answer only : 0 (4)

(b) $f(x) = \dfrac{3x^4+7x^2-5x}{2x^2}$

$$= \frac{3x^2}{2}+\frac{7}{2}-\frac{5x^{-1}}{2}$$ MA

$f'(x) = \dfrac{3}{2}.2x+\dfrac{5}{2}.x^{-2}$ M (Finding derivative)

$= 3x+\dfrac{5}{2x^2}$ A

No marks for $\dfrac{12x^3+14x-5}{4x}$ (4)

(c) Average Gradient $= \dfrac{f(b)-f(a)}{b-a}$

$= \dfrac{f(1)-f(-1)}{1-(-1)}$ M (Gradient formula)

$= \dfrac{16-0}{2}$ A (for 16 and 0)

$= 8$ CA

Beware of average of 2 derivatives (3)
[11]

QUESTION 4

(a) (1) $A = 50\,000(1,06)^{15}$ M (Sub into Compound formula)

$= 119\,827,9097$

$\approx$ R119 827,91 A

(2)

(2) Financial gain $= A - 50\,000$

$=$ R69 827,91 CA

(1)

(b) (1) $850\ 000 = \dfrac{x\left[1-\left(1+\dfrac{0,08}{12}\right)^{-360}\right]}{\dfrac{0,08}{12}}$

M (Annuity formula) A (-360)

$= x\{136,2834941\}$

$x = 6\ 236,998878$

$\approx 6\ 237$

i.e. Monthly instalment was R6 237

ALTERNATI VELY

$$850\,000\left(1+\frac{0,08}{12}\right)^{360} = \frac{x\left[\left(1+\frac{0,08}{12}\right)^{360}-1\right]}{\frac{0,08}{12}}$$

M (Annuity formula) A (360)

Using Compound formula : Max 2/4 (4)

(2) Outstanding Balance $= 850\ 000\left(1+\dfrac{0,08}{12}\right)^{12}$

$$-\frac{6237\left[\left(1+\frac{0,08}{12}\right)^{12}-1\right]}{\frac{0,08}{12}}$$ A

M (Subtraction of Annuity from Intial compounded)

$= 920\ 549,5808 - 77\ 650,18859$

$= 842\ 899,39$ A

(3)

ALTERNATIVELY

$$\frac{6237\left[1-\left(1+\frac{0,08}{12}\right)^{-348}\right]}{\frac{0,08}{12}}$$ A (-348)

M (Present value annuity)

$= 842\ 899,56$ A

(3) Paid off: $850\ 000 - 842\ 899,39$

M (Ans from 2 subtracted from 850000)

$= 7\ 100,61$

Interest $= 74\ 844 - 7\ 100,61$ M (74844 – previous ans)

$= \text{R}67\ 743,39$ CA

(3)

ALTERNATIVELY

$850\ 000 - 74844$ M

$= 775156$

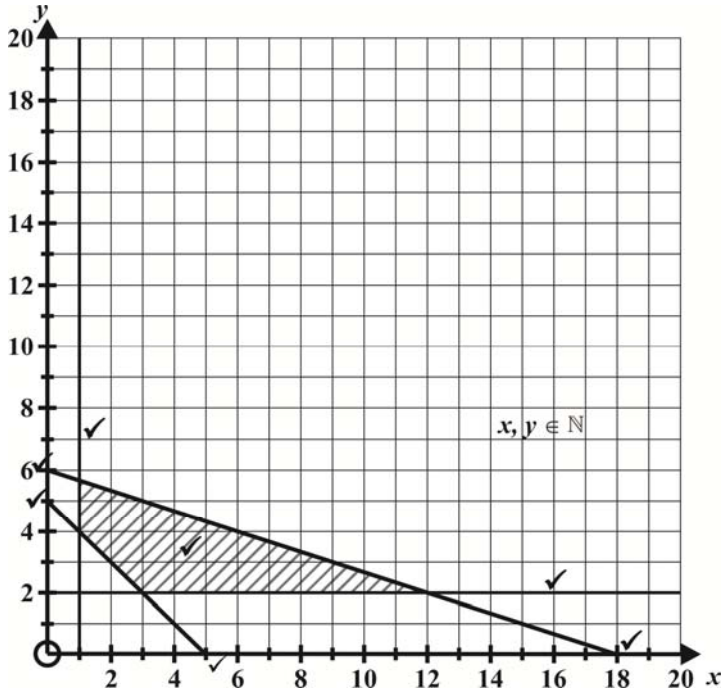
$842\ 899,56 - 775156$ M

$= 67\ 743,39$ CA

[13]

QUESTION 5

(a)



Feasible region CA (7)

(b) 5 A (1)

(c) Profit $= 500x + 500y$ A

$y = -x + \frac{P}{500}$

Max at (12; 2)

Profit $= 500 \times 12 + 500 \times 2$ M (Sub into Profit function)

$= R7\ 000$ A (3)

(d) Profit $= 200x + 600y$ A

$y = \frac{-x}{3} + \frac{P}{600}$ M

Same gradient as one of the constraints

(3 ; 5), (6 ; 4), (9 ; 3) (12 ; 2) AA (4)

[15]

74 marks

SECTION B

(2)

QUESTION 6

(a) (1) $f(73) = \left(\sqrt[12]{2}\right)^{73-49} \times 440$ M (Sub in 73)

$= 1\,760 \text{ Hz}$ A

(2) $\left(\sqrt[12]{2}\right)^{n-49} \times 440 = 3\,520$ M (= 3520)

$\left(\sqrt[12]{2}\right)^{n-49} = 8$ M (Solving)

$n - 49 = \log_{\sqrt[12]{2}}(8)$ M (Introducing logs)

$= 36$ A

$n = 85$ CA

i.e. 85th key (5)

(b) (1) $f(x) = 3x - 1$

Inverse: $x = 3y - 1$ M (Swopping x & y)

$3y = x + 1$

$f^{-1}(x) = \frac{x}{3} + \frac{1}{3}$ A

$g(x) = 2^x$

$g^{-1}(x) = \log_2 x$ M A (4)

(2) (i) $x < 1$ or $x > 3$ AA (2)

(ii) $0 < x < 1$ A A (2)

(iii) $x = 2$ or $x = 8$ A A (2)

(iv) $x > 2{,}1$ A (2)

A

– 1 if = included ONCE in whole question **[19]**

QUESTION 7

(a) $A\left(k; k^2\right)$ A

$f(x) = x^2$

$f'(x) = 2x$ A

$f'(k) = 2k$ CA

Eqn. of tangent:

$y - k^2 = 2k(x - k)$ M (Finding equation of str.line)

$y = 2kx - 2k^2 + k^2$ A

$= 2kx - k^2$

Starting with $A(-k; k^2)$: Max 3/5 (5)

(b) $y = 2mx - m^2$ A (1)

(c) At C: $2kx - k^2 = 2mx - m^2$ M (Equating lines) (5)

$$\begin{aligned} 2kx - 2mx &= k^2 - m^2 \quad \text{M (Isolating } x\text{)} \\ 2x(k-m) &= (k-m)(k+m) \quad \text{A} \\ x &= \frac{(k-m)(k+m)}{2(k-m)} \qquad k \neq m \quad \text{CA} \\ &= \frac{k+m}{2} \quad \text{A} \end{aligned}$$

(d) $x_D = \frac{k+m}{2} = x_C$ CA (x-coord of D) CA (conclusion)

$\therefore$ CD is parallel to the y-axis. (2)

[13]

QUESTION 8

(a)
$$\begin{aligned} f(x) &= a(x+6)(x+1)(x-2) \quad \text{A} \\ 24 &= a(6)(1)(-2) \quad \text{M (Sub (0;24) into expression)} \\ &= -12a \\ a &= -2 \\ f(x) &= -2(x+6)(x+1)(x-2) \quad \text{CA} \\ &= -2(x+6)(x^2 - x - 2) \\ &= -2(x^3 - x^2 - 2x + 6x^2 - 6x - 12) \quad \text{M (Expanding)} \\ &= -2(x^3 + 5x^2 - 8x - 12) \quad \text{A} \\ &= -2x^3 - 10x^2 + 16x + 24 \\ \therefore a &= -2,\ b = -10,\ c = 16 \text{ and } d = 24 \end{aligned}$$ (5)

(b) At D and E, $f'(x) = 0$ M (Derivative = 0)

$$\begin{aligned} -6x^2 - 20x + 16 &= 0 \quad \text{A (Derivative)} \\ 3x^2 + 10x - 8 &= 0 \\ (3x-2)(x+4) &= 0 \quad \text{A} \end{aligned}$$

$x = \frac{2}{3}$ or $x = -4$ CA

$f(-4) = -2(-4)^3 - 10(-4)^2 + 16(-4) + 24$ M (Sub negative value)

ALTERNATIVELY

$$\begin{aligned} f(-4) &= -2(2)(-3)(-6) \\ &= -72 \quad \text{A} \end{aligned}$$

$\therefore \text{D}(-4; -72)$ (6)

(c) $-4 + p = 0,\ -72 + q = 0$

$\therefore p = 4$ and $q = 72$ A A (2)

[13]

QUESTION 9

(a) $\underset{A}{B}\left(x;\underset{A}{4-\frac{x^2}{4}}\right)$ (2)

(b) Area $\Delta OBD = \frac{1}{2}\times(OD)(DB)$ M

$= \frac{1}{2}x\left(4-\frac{x^2}{4}\right)$ CA

$= 2x-\frac{x^3}{8},$ A $\quad x\in[0;4]$ (3)

(c) Max Area when $\frac{d\text{Area}}{dx}=0$ M

$2-\frac{3x^2}{8}$ A $=0$

$\frac{3x^2}{8}=2$ M (Solving)

$x^2=\frac{16}{3}$ A

$x=\frac{4\sqrt{3}}{3}$ CA (2,3 units)

$x\in[0;4]$ Only positive (5)

(d) Max Area $= 2\times\frac{4\sqrt{3}}{3}-\left(\frac{4\sqrt{3}}{3}\right)^3\div 8$

M (Sub ans from c into given formula from b)

$= \frac{16\sqrt{3}}{9}$ A (3,1) sq. units

ALTERNATIVELY

Max. area $= \frac{1}{2}\bullet\frac{4\sqrt{3}}{3}\left(4-\left(\frac{4\sqrt{3}}{3}\right)^2\div 4\right)$ M

$= \frac{16\sqrt{3}}{9}$ A

(2)

[12]

QUESTION 10

(a) $f(x) = -x^2 + 4x$

$A: -x^2 + 4x = 0$ M (y-int = 0)

$-x(x-4) = 0$

$x = 0$ or $x = 4$ A

$\therefore A(4;0)$ A

$x_E = \dfrac{-4}{2(-1)}$

$= 2$ A

$y_E = -2^2 + 4 \times 2$ M (Sub xE)

$= 4$ A

$\therefore E(2;4)$ (6)

(b) $y_G = 4 \times \left(\dfrac{3}{4}\right)^2$ M

$= \dfrac{9}{4}$

$x_B = x_A + \dfrac{3}{4} \times x_A$ M

$= 4 \times \dfrac{7}{4}$

$= 7$ A

$x_C = x_B + \dfrac{3}{4} \times AB$

$= 7 + \dfrac{3}{4} \times 3$ M

$= \dfrac{37}{4}$ A

$x_G = \dfrac{1}{2}(x_B + x_C)$

$= \dfrac{1}{2}\left(7 + \dfrac{37}{4}\right)$ M

$= \dfrac{65}{8}$

ALTERNATIVELY

$x_G = OA + AB + \dfrac{1}{2}BC$

$= 4 + \dfrac{3}{4} \times 4 + \dfrac{1}{2} \times \dfrac{9}{16} \times 4$ A

A(4) M M M

$= \dfrac{65}{8}$

$\therefore G\left(\dfrac{65}{8}; \dfrac{9}{4}\right)$ M (6)

(c) $y = a\left(x-\frac{65}{8}\right)^2+\frac{9}{4}$ A

$\text{B}:0 = a\left(7-\frac{65}{8}\right)^2+\frac{9}{4}$ M (Sub (7;0))

$\frac{81a}{64} = -\frac{9}{4}$

$a = -\frac{16}{9}$ A

$\therefore y = -\frac{16}{9}\left(x-\frac{65}{8}\right)^2+\frac{9}{4}$ CA

(4)

(d) Distance between x-int.s forms geometric sequence

$a = 4\ ;\ r = \frac{3}{4}$

$S_\infty = \frac{4}{1-\frac{3}{4}}$ M (Sub into Sum to infinity)

$= 16$ A

$\therefore$ parabolas will NOT fit on OH. CA

(3)

[19]

76 marks

Total: 150 marks